PROVA 1 - CÁLCULO 1 - 14/06/22

①

Problema 1 -

a - $\lim_{x - 1} \sqrt[3]{\frac{x^3+1}{x+1}}$, Podemos escrever

$$\frac{x^3+1}{x+1} = \frac{\cancel{(x+1)} \; \cancel{(x+1)^2} \; (x^2-x+1)}{\cancel{x+1}}$$, simplifica

ficando: $x^2 - x + 1$, assim,

$$\lim_{x \to -1} \sqrt[3]{\frac{x^3+1}{x+1}} = \lim_{x \to 1} \sqrt[3]{x^2-x+1} = \sqrt{3}$$

b - $\lim_{x \to 0} \frac{\sin 3x}{x}$, definindo $u = 3x$

$\frac{u}{3} = x$

$x \to 0$

$u \to 0$

$$\lim_{u \to 0} \frac{\sin u}{\frac{u}{3}} = \lim_{x \to 0} 3 \frac{\sin u}{u}$$

$$3 \lim_{u \to 0} \frac{\sin u}{u} \to 1 = 3$$

b) $\lim_{x \to 0} \frac{tg(3x)}{\sin(4x)}$, usando $tg\,x = \frac{\sin x}{\cos x}$

Assim:

$$\lim_{x\to 0} \frac{\sin 3x}{\cos(3x)} \cdot \frac{1}{\sin(4x)} = \underbrace{\lim_{x\to 0} \frac{1}{\cos(3x)}}_{1 \to \cos 0 = 1} \cdot \lim_{x \to 0} \frac{\sin 3x}{\sin 4x}$$

Assim:

$$\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} \underbrace{\frac{x\; 3x \times 4x}{x\; 3x \times 4x}}_{1}$$, Assim:

$$\lim_{x\to 0} \frac{\sin 3x}{3x} \cdot 3x \cdot \lim_{x\to 0} \frac{4x}{4x \sin 4x}$$,

Tomando 1: $u = 3x \Rightarrow \frac{u}{3} = x \to \begin{matrix} x \to 0 \\ u \to 0 \end{matrix}$

2: $v = 4x \Rightarrow \frac{v}{4} = x \quad \begin{matrix} x \to 0 \\ v \to 0 \end{matrix}$

$$\lim_{u\to 0} \frac{\sin u}{u} \cdot 3x \cdot \lim_{v \to 0} \frac{v}{4x \sin v}$$,

$$\underbrace{\lim_{u\to 0} \frac{\sin u}{u}}_{\to 1} \cdot \underbrace{\lim \frac{v}{\sin v}}_{\to 1} \cdot \frac{3x}{4x} = \frac{3}{4}$$

Questão 1-

d) $\lim\limits_{x\to 0} \dfrac{3x^2}{tg x \cdot \sin x} \Rightarrow tg(x) = \dfrac{\sin(x)}{\cos x}$

$= \lim\limits_{x\to 0} \dfrac{\cos x}{\sin x} \dfrac{3x^2}{\sin x} = 3 \lim \dfrac{x \cdot x \cdot \cos x}{\sin x \cdot \sin x} =$

$= 3 \cdot \lim\limits_{x\to 0} \dfrac{x}{\sin x} \cdot \lim\limits_{x\to 0} \dfrac{x}{\sin x} \cdot \lim\limits_{x\to 0} \cos x$

$\to 1 \quad \to 1 \quad \to 1$

$\cos 0 = 1$

$\therefore \boxed{= 3}$

Questão 2

Sabemos que:

$f'(x) = \lim\limits_{x\to p} \dfrac{f(x) - f(p)}{x - p}$, se $x = p$, logo, e $f(x) = y$

$f'(x) = \dfrac{\overbrace{f(x)}^{y} - f(p)}{x - p} \Rightarrow y - f(p) = f'(x)(x - p)$

Assim:

a) $f(x) = \sqrt{x}$ em $p = 9$,

$f'(x) = \dfrac{1}{2} \dfrac{1}{\sqrt{x}}$ (conhecemos)!

Usando:

$$y - f(p) = f'(x)(x - p) \quad \text{temos:}$$

$$y - \sqrt{2} = \frac{1}{2}\frac{1}{\sqrt{x}}(x-2), \quad \text{logo}$$

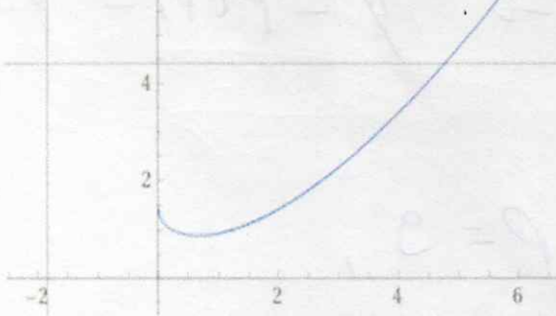
$$\boxed{y = \frac{1}{2\sqrt{x}}(x-2) + \sqrt{2}}$$

b) $x^2 - x$, $p = 1$.

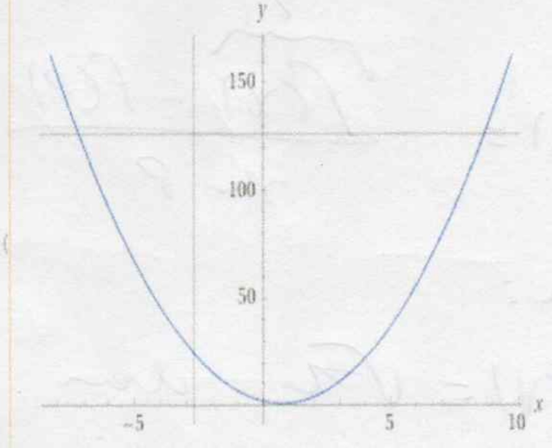
$$y - 1 = (2x-1)(x-1);$$

$$\boxed{y = 2x^2 - 3x + 2}.$$

A)



B)



Questão 3 f', f''

a) $f(x) = 5x^2 - x^{-3}$

$f' = 10x + 3x^{-4}$

$f'' = 10 - 12x^{-5}$

b) $f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$f''(x) = \ln(x)$

Questão 4

a) $f(x) = x \ln(2x+1)$

$f'(x) = 1 \cdot \ln(2x+1) + x \cdot \frac{1}{2x+1} \cdot 2$

$\boxed{f'(x) = \ln(2x+1) + \frac{2x}{2x+1}}$

b) $y = [\ln(x^2+1)]^3$, pelas propriedades dos logaritmos temos:

$y = 3\ln(x^2+1)$ ∴

$y' = \frac{3 \cdot 2x}{x^2+1} = \frac{6x}{x^2+1}$

Questão 3

c) $f(x) = e^{-x^2} + \ln(2x+1)$

$$\boxed{f'(x) = -2xe^{-x^2} + \frac{2}{2x+1}}$$

d) $f(T) = \dfrac{T e^{2T}}{\ln(3T+1)}$

$$f'(T) = \frac{(1 \cdot e^{2T} + 2T e^{2T}) \cdot \ln(3T+1) - \frac{3Te^{2T}}{3T+1}}{2\ln(3T+1)}$$

$$f'(t) = e^{2t}\left[\frac{1+2t}{\ln(3T+1)} - \frac{3t}{(3T+1)(2\ln(3T+1))}\right]$$

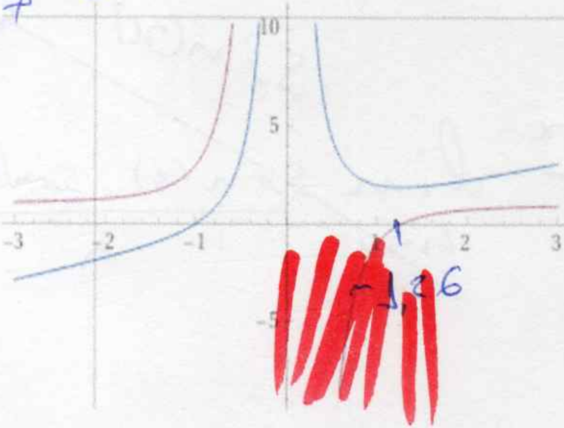
5) GRÁFICOS

A)

$$f(x) = x + \frac{1}{x^2}$$

$$f'(x) = 1 - \frac{2}{x^3}$$

$x \neq 0$



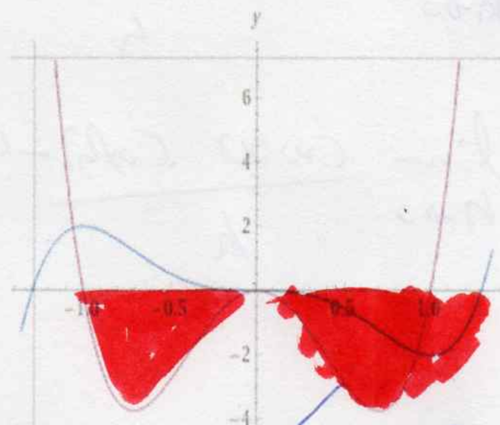
$0 < x \leq 1,26$

Função decrescente

B)

$$f(x) = 3x^5 - 5x^3$$

$$f'(x) = 15x^4 - 15x^2$$



Função decrescente na zona vermelha

$-1 \leq x \leq 1$

Questão bônus: $f(x) = \cos(x) \to f'(x) = -\sin(x)$

Sabemos que: $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$, assim:

$f'(x) = \lim_{h \to 0} \dfrac{\cos(x+h) - \cos(x)}{h}$, usando a soma dos arcos do cosseno, temos:

$$f'(x) = \lim_{h \to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x)\,\cos(h) - \cos x}{h} - \cancel{\lim_{h \to 0} \frac{\sin(x)\,.\,\sin(h)}{h}} \to \sin(x) \to 1$$

Assim:

$$f'(x) = \lim_{h \to 0} \frac{\cos(x)(\cos h - 1)}{h} \cdot \left(\frac{\cos h + 1}{\cos h + 1}\right) - \sin(x)$$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x)(-\sin^2 h)}{h(\cos h + 1)} - \sin x =$$

$$f'(x) = -\lim_{h \to 0} \frac{\cos x \sin h \sin h}{h(\cos h + 1)} - \sin(x)$$

$$= f'(x) = -\lim_{h \to 0} \frac{\cos(x)\sin h}{\cos h + 1} \cdot \cancel{\lim \frac{\sin h}{h}} \to 1 - \sin x$$

Tomando o limite:

$$f'(x) = -\cancel{\frac{\cos x \cdot \sin 0}{\cos 0 + 1}} \to 0 - \sin x = \boxed{-\sin(x)}$$