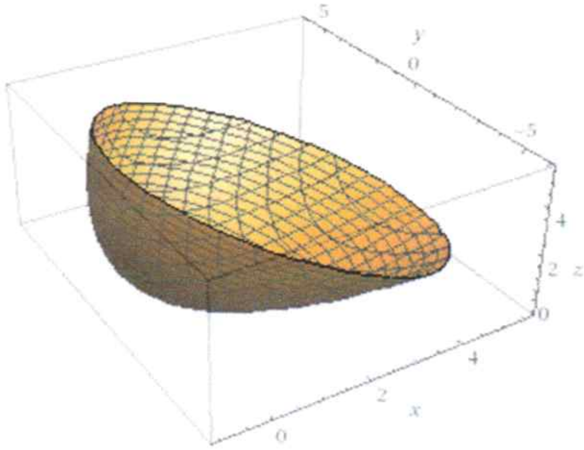
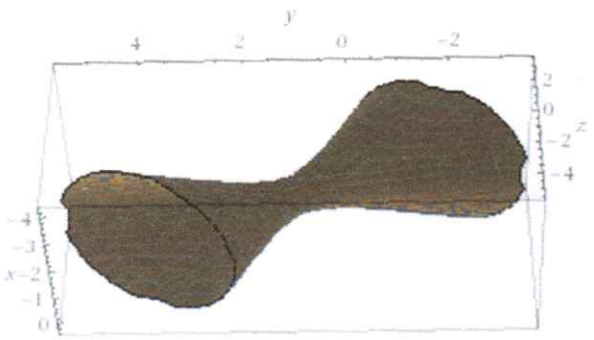
GABARITO PROVA 1

a) ~~ELIP~~ paraboloide elíptico.



b) hiperboloide de uma folha.



Questão 2 :

a equação geral dos planos é dado por

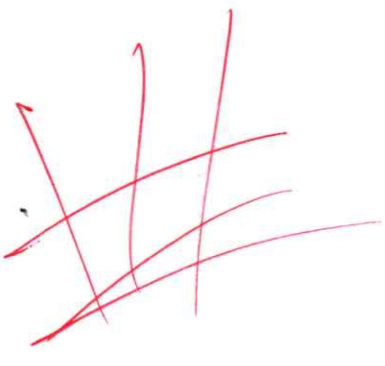
$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

dado os pontos $P(3,-1,2)$ e $P_1=(2,1,4)$ e $P_2=(-3,-1,7)$, onde $P \perp P_1$ e $\perp P_2$.

a equação do plano no ponto P é dado por.

$a(x-3)+b(y+1)+c(z-2)$. basta calcular distancia do ponto e reta e encontra a distância solicitada no problema.

$$d_1 = \frac{\|\vec{V} \times \vec{W}\|}{\|\vec{V}\|}$$



Questão 3 ③

a) $\underbrace{\frac{\partial}{\partial x}}_{I}, \underbrace{\frac{\partial}{\partial y}}_{II}, \underbrace{\frac{\partial}{\partial z}}_{III}, \underbrace{\frac{\partial}{\partial w}}_{IV}$ em $f(x,y,z,w) = \frac{3x^5 z w^4}{\sqrt{w^2x^3+y^3z^2}} \cot g(xyzw)$

Use-se a regra do produto. $\partial 1 \cdot 2 + 1 \cdot \partial 2$

$$I \Rightarrow \frac{\partial}{\partial x}\left(\overbrace{\frac{3x^5 z w^4}{\sqrt{w^2x^3+y^3z^2}} \cot g(xyzw)}\right) = \frac{\partial}{\partial x}\left[\left(\frac{6x^5w^4}{\sqrt{w^2x^3+y^3z^2}}\right)\left(\cot g(xyzw)\right)\right]$$

$$\frac{\partial}{\partial x} f(x,y,z,w) = \frac{3w^4x^4\left((7w^2x^3+10y^3z^2)\cot g(wxyz) - 2w\,xyz\,(w^2x^3+y^3z^2)\operatorname{cossec}^2(xyzw)\right)}{(w^2x^3+y^3z^2)^{3/2}}$$

$$\frac{\partial}{\partial y} f(x,y,z) = -\frac{3w^4x^5z\left(2wx(w^2x^3+y^3z^2)\operatorname{cossec}^2(xyzw) + 3y^2z\cot g(xyzw)\right)}{(w^2x^3+y^3z^2)^{3/2}}$$

$$\frac{\partial}{\partial z} f(x,y,z,w) = -\frac{6w^4x^5y\left(wx(w^2x^3+y^3z^2)\operatorname{cossec}^2(xyzw) + y^2z\cot g(xyzw)\right)}{(w^2x^3+y^3z^2)^{3/2}}$$

$$\frac{\partial}{\partial w} f(x,y,z,w) = -\frac{6w^3x^5\left(w\,xyz\,(w^2x^3+y^3z^2)\operatorname{cossec}^2(xyzw) - (3w^2x^3+4y^3z^2)\cot g(xyzw)\right)}{(w^2x^3+y^3z^2)^{3/2}}$$

Questão 3. b

$$f(x,y,z,w) = \frac{xy^2z^3w}{1+x^2+y^4+z^6+w^8}$$

$$\frac{\partial f}{\partial x} = \frac{wy^2z^3(w^8-x^2+y^4+z^6+1)}{(1+x^2+y^4+z^6+w^8)^2}$$

$$\frac{\partial f}{\partial y} = \frac{2wxyz^3(w^8+x^2-y^4+z^6+1)}{(1+x^2+y^4+z^6+w^8)^2}$$

$$\frac{\partial f}{\partial z} = \frac{3wxy^2z^2(w^8+x^2+y^4-z^6+1)}{(1+x^2+y^4+z^6+w^8)^2}$$

$$\frac{\partial f}{\partial w} = \frac{xy^2z^3(-7w^8+x^2+y^4+z^6+1)}{(1+x^2+y^4+z^6+w^8)^2}$$

Questão 5

a)

$$\nabla f = \nabla\left(\frac{y}{\sqrt{x^2+y^2+z^2}} + \exp\left(-\frac{1}{2}(x^2+y^2-z^2)\right)\right)$$

$$= \Bigg(-x\,e^{-1/2(x^2+y^2-z^2)} - \frac{xy}{(x^2+y^2+z^2)^{3/2}}\,\hat{i} +$$

$$\frac{x^2+z^2}{(x^2+y^2+z^2)^{3/2}} - y\,e^{-1/2(x^2+y^2-z^2)}\,\hat{j} +$$

$$z\,e^{-1/2(x^2+y^2-z^2)} - \frac{yz}{(x^2+y^2+z^2)^{3/2}}\,\hat{k}\Bigg)$$

$$\nabla^2 f =$$

$$\frac{e^{-1/2(x^2+y^2-z^2)}\left(e^{z^2/2}\sqrt{x^2+y^2+z^2}\,(x^4+x^2(2y^2+2z^2-1)+y^4+y^2(2z-1)+z^2(z^2-1)) - 2y\,e^{1/2(x^2+y^2)}\right)}{(x^2+y^2+z^2)^{3/2}}$$

Questão 5 b

Gradiente

$$\nabla f = \frac{1}{2}\left(yz(x^2-z^2)\cot(xyz) - 2x\right)\operatorname{cossec}(xyz)\,\hat{i} +$$

$$+\frac{1}{2}xz(x^2-z^2)\cot(xyz)\operatorname{cossec}(xyz)\,\hat{j} +$$

$$+\frac{1}{2}\left(xy(x^2-z^2)\cot(xyz)+2z\right)\operatorname{cossec}(xyz)\,\hat{k}$$

e

Laplaciano:

$$\nabla^2 f =$$

$$-\frac{1}{4}(x^2-z^2)\left(x^2(y^2+z^2)+y^2z^2\right)\left(\cos(2xyz)+3\right)\operatorname{cossec}^3(xyz)$$

Questão 6

a) divergente

$$\vec{\nabla} \cdot \vec{F} = \frac{3z^2}{y^2} + 4yz$$

Rotacional.

$$\vec{\nabla} \times \vec{F} = \frac{6z}{y}\hat{i} + 4xy + \sin(x)\hat{j} - 4xz\hat{k}$$

b) divergente

$$\vec{\nabla} \cdot \vec{F} = \frac{\cos\theta}{\rho} - \rho + \frac{z}{z}$$

Rotacional

$$\vec{\nabla} \times \vec{F} = \left(0, \ z - \frac{\rho}{z^2}, \ \frac{\sin\theta}{\rho}\right)$$

c) $$\vec{\nabla} \cdot \vec{F} = 4\sin(2\theta)\cos(2\theta)\tan(\varphi) + \sin^2(2\theta)\cot(\theta)\tan(\varphi) + $$
$$+ 3\sin(2\theta) - \frac{\operatorname{cossec}\theta \sec\theta}{r^2}.$$

$$\vec{\nabla} \times \vec{F} = \frac{-\frac{\varphi \operatorname{cossec}\theta}{r} - \frac{\theta \tan\theta \sec\theta}{r} - r\sin^2(2\theta)\operatorname{cossec}(\theta)\sec^2\theta}{r}\hat{r} +$$
$$+ 0\hat{\theta} +$$
$$\sin^2(2\theta)\tan(\varphi) + \frac{r\sin^2(2\theta) + g(\varphi) - 2r\cos(2\theta)}{r}\hat{\varphi}$$